2024 AP Calculus (BC) Summer Assignment

This packet is a review of concepts you need to know from Calculus AB (Calc 1). It is to be done NEATLY and on a SEPARATE sheet of paper. Use your discretion as to whether you should use a calculator or not. When in doubt, think about whether you would have used the GC in Calc AB – that should guide you! Points will be awarded only if the correct work is shown, and that work leads to the correct answer. Have a great summer and I am looking forward to seeing you in September. ☺

This assignment is due the first day of class. If you have questions over the summer, you can email me at ccanonaco@manasquan.k12.nj.us. I check email about once per week in the summer and will get back to you.

LATE work will not be accepted!

Part I: Unlimited and Continuous!

1. Find the limits, if they exist using any method.

a. $\lim_{x\to4}\dfrac{2x^3-7x^2-4x}{x-4}$ b. $\lim_{x\to9}\dfrac{\sqrt{x}-3}{9-x}$ c. $\lim_{x\to3}\dfrac{\left|x^2-9\right|}{4x-12}$

d. $\lim_{x\to0}\dfrac{1-\cos x}{x}$ e. $\lim_{x\to0}\dfrac{\tan(\pi x)}{x}$ f. $\lim_{x\to\infty}\dfrac{2x^3-7x^2-4x}{4-x^3}$

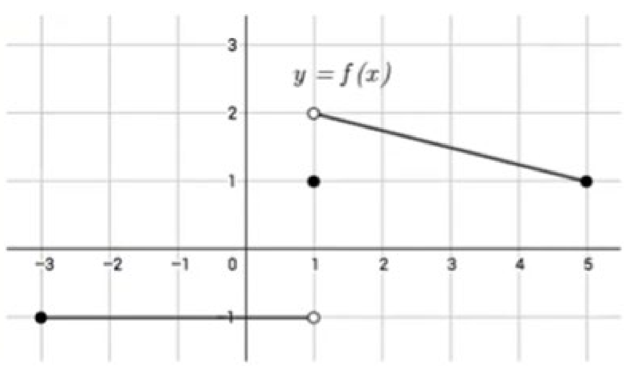
g. $\lim_{x\to-\infty} e^x$ h. $\lim_{x\to\infty} e^x$ i. $\lim_{x\to0^+}\ln x$

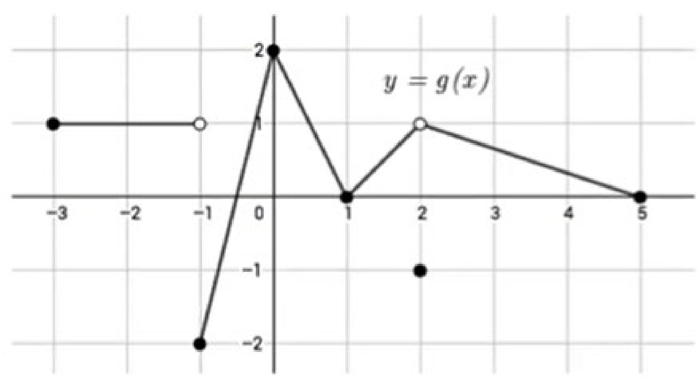
j. $\lim_{x\to-\infty}\dfrac{\sqrt{16x^2+2x-1}}{9+5x}$ k. $\lim_{h\to0}\dfrac{\cos\left(\dfrac{\pi}{3}+h\right)-\dfrac{1}{2}}{h}$ l. $\lim_{x\to8}\dfrac{\sqrt[3]{x}-2}{x-8}$

2. Explain why each function is discontinuous and determine if the discontinuity is removable or non-removable.

a. $g(x)=\begin{cases}2x-3, & x<3\\ -x+5, & x\ge3\end{cases}$ b. $h(x)=\dfrac{\sqrt{x^2-10x+25}}{x-5}$

3. Evaluate each limit based on the graph below of $f(x)$ & $g(x)$.





a. $\lim_{x\to1} f(x)g(x)$ b. $\lim_{x\to2} f(g(x))$

c. $\lim_{x\to0}\dfrac{f(x+2)}{g(x)}$ d. $\lim_{x\to1^-} g(f(x))$

4. Consider the function $f(x)=\begin{cases}x^2+kx & x\le5\\ 5\sin\left(\dfrac{\pi}{2}x\right) & x>5\end{cases}$. Find the value of k that will make the function continuous.

Part II: Designated Deriving!

1. Find the derivative.

   a. $y = \ln(1+e^x)$ b. $y = \csc(1+\sqrt{x})$ c. $y = \sqrt[7]{x^3 - 4x^2}$

   d. $f(x) = (x+1)e^{3x}$ e. $f(\theta) = \cos^2(\theta) - \sin^2(\theta)$ d. $y = \dfrac{4-x^2}{x^3}$

2. Consider the function $f(x) = \sqrt{x-2}$. On what intervals are the hypotheses of the Mean Value Theorem satisfied?

3. If $xy^2 - y^3 = x^2 - 5$, then $\dfrac{dy}{dx} =$

4. If $y = \cos\left(x^2\right)$, then $\dfrac{d^2y}{dx^2} =$

5. The distance of a particle from its initial position is given by $s(t) = t - 5 + \dfrac{9}{(t+1)}$, where s is feet and t is minutes. Find the velocity at t = 1 minute in appropriate units.

6. Use the table to answer the questions below.

x	$f(x)$	$g(x)$	$f'(x)$	$g('x)$
1	4	2	5	½
3	7	-4	$\frac{3}{2}$	-1

   a. The value of $\dfrac{d}{dx}(f \cdot g)$ at x = 3 is

   b. The value of $\dfrac{d}{dx}\left(\dfrac{f}{g}\right)$ at x = 1 is

7. Use the table below to find the value of the first derivative of the given functions for the given value of x.

x	$f(x)$	$g(x)$	$f'(x)$	$g('x)$
1	3	2	0	$\frac{3}{4}$
2	7	-4	$\frac{1}{3}$	-1

   a. $\dfrac{d}{dx}[f(x)]^2$ at x = 2 is

   b. $\dfrac{d}{dx}f(g(x))$ at x = 1 is

   c. $\left(f^{-1}\right)'(3)$ d. $\left(g^{-1}\right)'(-4)$

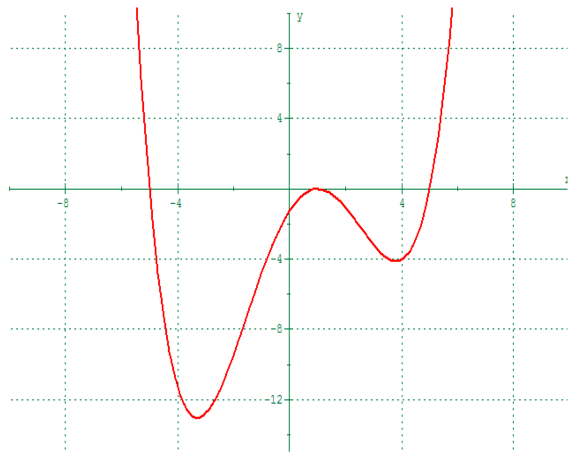
Part III: Derived and Applied!

1. Find the absolute extrema of the function on the closed interval: $j(x) = x\ln x, \quad [1,4]$

2. Completely analyze the function by discussing all intercept(s), asymptotes, extrema, regions where the function is increasing and decreasing, points of inflection and regions of concavity. Then, sketch the function. Justify using the first and second derivatives.

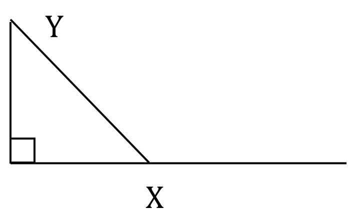
$$f(x) = \frac{6}{x^2+3} \text{ given } f'(x) = \frac{-12x}{(x^2+3)^2}, \quad f''(x) = \frac{36(x^2-1)}{(x^2+3)^3}$$

3. For the given graph of $f'(x)$ answer the following questions:



a. intervals on which $f(x)$ is increasing
b. intervals on which $f(x)$ is decreasing
c. relative maximum of function at $x =$
d. relative minimum of function at $x =$
e. points of inflection for $f(x)$ at $x =$
f. intervals on which $f(x)$ concave up
g. intervals on which $f(x)$ concave down
h. Sketch a graph of $f(x)$ on the same coordinate plane.

4.



A ladder 15 feet long is leaning against a building so that end X is on level ground and end Y is on the wall as shown in the figure. X is moved away from the building at a constant rate of ½ foot per second.

a. Find the rate in feet per second at which the length OY is changing when X is 9 feet from the building.
b. Find the rate of change in square feet per second of the area of triangle XOY when X is 9 feet from the building.

Part IV: Integral to Your Success!

1. $\int_{-8}^{-1} \frac{x - x^2}{2\sqrt[3]{x}} dx$

2. $\int_{-\pi/6}^{\pi/6} \sec^2 x dx$

3. $\frac{d}{dx} \int_1^x \sqrt[4]{t}\, dt$

4. $\int \frac{x^3}{\sqrt{1+x^4}} dx$

5. $\int \frac{\csc^2 x}{\cot^3 x} dx$

6. $\int \sqrt{\tan x} \sec^2 x dx$

8. What is the average value of $y = x^3\sqrt{x^4 + 9}$ on the interval [0, 2]?

9. The function *f* is continuous on the closed interval [1, 9] and has the values given in the table. Using the subintervals [1, 3], [3, 6], and [6, 9], what is the value of the trapezoidal approximation of $\int_1^9 f(x)dx$?

x	1	3	6	9
$f(x)$	15	25	40	30

10. The table below provides data points for the continuous function $y = h(x)$.

x	0	2	4	6	8	10
$h(x)$	9	25	30	16	25	32

Use a right Riemann sum with 5 subdivisions to approximate the area under the curve of $y = h(x)$ on the interval [0, 10].

11. Calculator Allowed: The velocity of a particle that moves along the x-axis at any time $t \geq 0$ is given by $v(t) = e^{1-2t} - \frac{1}{e}$. At $t = 0$, the particle is at $x = 1$.

a. Find the acceleration of the particle at $t = 2$.

b. Is the speed of the particle increasing at $t = 2$? Give a reason for your answer.

c. Find all values of t where the particle changes direction. Justify your answer.

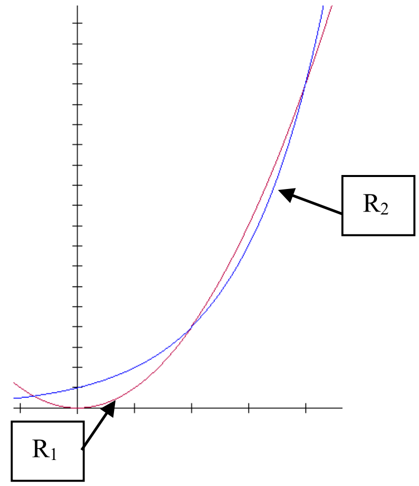
d. Find the position of the particle at $t = 2$.

Part V: Apply Those Integrals!

1. Find the particular solution to the differential equation $\frac{dy}{dx} = y\sin x$ at the point $(0,1)$.

2. The shaded regions, R_1 and R_2 shown above are enclosed by the graphs of $f(x) = x^2$ and $g(x) = 2^x$.

    a. Find the x- and y-coordinates of the three points of intersection of the graphs of f and g.
    b. Without using absolute value, set up an expression involving one or more integrals that gives the total area enclosed by the graphs of f and g. Do not evaluate.
    c. Find the volume when the region R_1 is rotated around the x-axis.
    d. Find the volume when the region R_1 is rotated around the line $x = -1$.



3. Let R be the region in the first quadrant under the graph of $y = \frac{1}{\sqrt{x}}$ for $4 \le x \le 9$.

    a. Find the area of R.
    b. Find the volume of the solid whose base is the region R and whose cross sections cut by planes perpendicular to the x-axis are squares.